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Reliability Analysis of a Biogas Plant having two Dissimilar Units

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Abstract: **A biogas system with two dissimilar production units is considered. mathematical modelling is carried out for the system to calculate the reliability and mean time to system failure of the system. Two models (parallel redundant and standby redundant) are given. Formulation is done using a simple probabilistic approach and the supplementary variable technique. The problem is solved using simple methods for solving differential equations. Expressions for profit function are given.**

Keywords: **biogas system, mathematical modelling, solving differential equations.**

1. INTRODUCTION

Reliability technology has been used extensively in electronics systems design. Power systems, mechanical systems design, aerospace and military combat problems. During the past 25 years many research papers and books have been published discussing various facets of reliability technology. Recently Singh [1-3] suggested some applications of reliability technology to agriculture and allied fields, e.g. the sugar industry, fertilizer industry and energy modelling. Kumar *et al*. [4, 5] applied the technology to the sugar and paper industries. These recent developments impressed that reliability technology may be used widely in non-conventional energy resources development. It may be used for designing both biogas and biomass plants. Since biogas from animal waste or municipal waste may be used as cooking gas, for lighting purposes, to run small industrial plants, etc., it requires more study for development. In this paper an attempt is made to extend further the biogas plant models discussed in [3]. Mathematical formulation and solution of the governing equations is carried out by simple methods.

2. THE MODELS

Consider an energy producing plant having two closed tanks and using animal waste for energy (gas) production. The tanks have separate inlets and outlets and are similar in design, having different capacities. The two models under consideration are given below.

Model I

This represent a repairable parallel redundant system where both units (tanks) are operating simultaneously. The system also works, but in reduced capacity, when one unit (tank) fails due to one or other reason.

Model II

This represents a repairable standby system where one unit (tank) is active and the other is in standby. When the active unit fails it is replaced by the standby unit, i.e. the gas (energy) supply channel from the standby unit is open.

The operating and standby tanks may fail simultaneously. Failures may occur due to leakage from the tank(s), due to low temperature of the atmosphere, due to sudden snow fall and the normal failure due to reasons of waste material. By repair of the system we mean overcoming the difficulties caused due to the aforesaid reasons. The transition diagrams for the two systems are shown in Fig. 1.

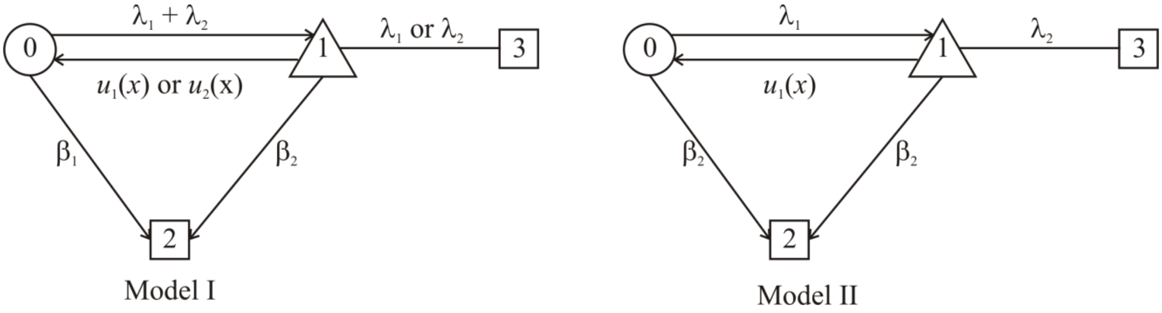

Fig. 1

3. ASSUMPTIONS AND NOTATION

1. Failures occurring in the system are statistically independent.

2. Failure rates are constant.

3. The entire system can fail due to sudden critical changes in the atmosphere.

4. Sudden critical failures may occur when either both units are good or when one unit is good.

5. System units (tanks) are of similar design, but of different capacity.

6. the repaired system works like new.

⓪ the plant is operating at full capacity.

① the plant is operating on the standby unit at full capacity

△1 the plant is operating at reduced capacity with only one unit

2 the plant is in the failed state due to critical failures

λ_1, λ_2 constant failure rates of units one and two

$u_1(x), u_2(x)$ repair rates of units one and two

β_1 constant critical failure rate when both units are operating

β_2 constant critical failure rate when one unit is operating

$P_i(t)$ probability that due to failure one unit is in repair and the repair time lies within the interval $(x, x+dx)$

We use λ for λ_1 or λ_2 and $u(x)$ for $u_1(x)$ or $u_2(x)$, whichever is applicable, when formulating for model I. Limits for integration are taken from 0 to ∞.

4. FORMULATION AND SOLUTION OF EQUATIONS

The governing differential equations associated with the transition diagrams are as follows.

Model I

$$dP_0(t)/dt = -(\lambda_1 + \lambda_2 + \beta_1)P_0(t) + \int P_1(x,t)u(x)dx, \tag{1}$$

$$\partial P_1(x,t)/\partial x + \partial P_1(x,t)/\partial t = -(\lambda + u(x) + \beta_2)P_1(x,t) + (\lambda_1 + \lambda_2)P_0(t) \tag{2}$$

$$dP_2(t)/dt = \beta_1 P_0(t)\beta_2 \int P_1(x,t)\,dx \tag{3}$$

$$dP_3(t)/dt = \lambda \int P_1(x,t)\,dx\,, \tag{4}$$

with boundary and initial conditions

$$P_1(0,t) = (\lambda_1 + \lambda_2)P_0(t)\,, \tag{5}$$

$$P_1(x,0) = P_2(0) = P_3(0) = 0\,, \tag{6}$$

$$P_0(t) = \begin{cases} 1 & \text{when}\; t = 0, \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

Model II

$$dP_0(t)/dt = -(\lambda_1 + \beta_2)P_0(t) + \int P_1(x,t)u_1(x)\,dx\,, \tag{8}$$

$$\partial P_1(x,t)/\partial x + \partial P_1(x,t)/\partial t = -\left(\lambda_2 + u_1(x) + \beta_2\right)P_1(x,t) + \lambda_1 P_0(t)\,, \tag{9}$$

$$dP_2(t)/dt = \beta_2 P_0(t) + \beta_2 \int P_1(x,t)\,dx\,, \tag{10}$$

$$dP_3(t)/dt = \lambda_2 \int P_1(x,t)\,dx\,, \tag{11}$$

with boundary condition

$$P_1(0,t) = \lambda_1 P_0(t)\,, \tag{12}$$

while initial conditions are the same as for model I.

equations (2) and (9) are Lagrange type linear partial differential equations of first order while the other differential equations are linear differential equations of first order. Solving the above system of equations we get the following.

Model I

$$P_0(t) = \left[1 + B(x) + \lambda t \int B(x) + \beta_1 + \beta_2 \int B(x)\,dx\right]^{-1},$$

$$P_1(x,t) = P_0(t)B(x),$$

$$P_2(t) = \left[\beta_1 + \beta_2 \int B(x)\,dx\right]P_0(t),$$

$$P_3(t) = \left[\lambda t \int B(x)\,dx\right]P_0(t),$$

where

$$B(x) = (\lambda_1 + \lambda_2)\left[1 + \int \exp\left(\lambda x + \beta_2 x + \int u(x)\,dx\right)dx\right]\exp\left(-\lambda x - \beta_2 x - \int u(x)\,dx\right).$$

Model II

$$P_0(t) = \left[1 + B_1(x) + \lambda_2 t \int B_1(x)dx + \beta_2 + \beta_2 \int B_1(x)\,dx\right]^{-1},$$

$$P_1(x,t) = B_1(x)P_0(t),$$

$$P_2(t) = \left(1 + \int B_1(x)dx\right)\beta_2 P_0(t),$$

$$P_3(t) = \left[\lambda_2 t \int B_1(x)\,dx\right]P_0(t),$$

where

$$B_1(x)=\lambda_1\left[1+\int \exp\left(\lambda_2 x+\beta_2 x+\int u(x)dx\right)dx\right]\exp\left(-\lambda_2 x-\beta_2 x-\int u(x)dx\right).$$

5. RESULTS

The reliability functions $R(t)$ for both the systems are given by

$$R_I(t)=P_0(t)+P_1(t)=P_0(t)+\int P_1(x,t)dx$$

$$=\left[1+\int B(x)dx\right]P_{0(t)}.$$

$$R_{II}(t)=\left[1+\int B_1(x)dx\right]P_0(t),$$

where $B(x), B_1(x), P_0(t)$ are as given in Section 4 above, and mean time to system failure (MTSF) may be evaluated from the following relations

$$(\text{MTSF})_I=\int_0^\infty R_I(t)dt,$$

$$(\text{MTSF})_{II}=\int_0^\infty R_{II}(t)dt.$$

In model II if we start with the second unit, the results are obtained simply by replacing λ_1 and $u_1(x)$ by λ_2 and $u_2(x)$, respectively. If $\lambda_1=\lambda_2=\lambda, u(x)=u$ (constant repair rate),we get the results obtained in [3] after minor manipulations.

6. PROFIT FUNCTION

Consuming biogas, if the consumer saves the money which was to be expanded for commercial gas then we may assume this saving as the revenue cost. Let C_1, C_2 be the revenue costs per unit time and C_3, C_4 be the service costs per unit time for models I and II, respectively, then the expected profit function $G_i(t)$, i = I, II during the interval (0, *t*] is given by

$$G_I(t)=C_1\int_0^t R_I(t)dt-C_3 t,$$

$$G_{II}(t)=C_2\int_0^t R_{II}(t)dt-C_4 t.$$

Making a profit table one may guess about the suitability of the model.

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